3. ARUTYUNYAN N.KG. and MKHITARYAN S.M., Certain contact problems for a half-space reinforced by elastic inserts. PMM, 36, 5, 1972.
4. ALEKSANDROV V.M. and MKHITARYAN S.M., Contact Problems for Bodies with Thin Coatings and Laminations. Nauka, Moscow, 1983.
5. PAVLENKO A.V., Use of the asymptotic method in solving three-dimensional problem of the theory of elasticity for composite materials. Izv. Akad. Nauk SSSR, MTT, 3, 1980.
6. GALIN L.A., Contact Problems of the Theory of Elasticity and Viscoelasticity. Nauka, Moscow, 1980.
7. BAKER G.A. and GRAVES-MORRIS P. Padé Approximants. Cambridge University Press, 1984.

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THE COMBINED PROBLEM OF THERMOELASTIC CONTACT BETWEEN TWO PLATES THROUGH A HEAT CONDUCTING LAYER*

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The problem of the contact between two plates under the action of a force and temperature field is formulated. It is assumed that when the plates are deformed, the conditions of heat exchange between them also change. The equations of motion and heat conduction of the thermoelastic plates, as well as the equations of heat conduction of the heat conducting layer are derived by expanding the three-dimensional equations in series in Legendre polynomials. The equations of the n-th approximation are constructed and the equations of the first approximation are studied in detail.

1. Formulation of the problem. We consider two plates (*1* and *2*) of arbitrary contour and constant thickness h_1 and h_2, respectively, situated, in an initial undeformed state, a distance h_0 apart. We shall assume that h_0 is commensurable with the flexures of the plates, and we will assume the flexures to be small. A heat-conducting medium is enclosed between the plates. The medium does not resist their deformation, and heat exchange within it is due to its thermal conductivity. Let Ω_γ $(\gamma = 1, 2)$ be the regions occupied by the median surfaces of the plates, $\partial\Omega_\gamma$ their boundaries, Ω_γ^+ and Ω_γ^- the upper and lower surfaces of the plates, and Γ_γ the side surfaces.

The thermodynamic state of the system, including the plates and heat conducting layer, is defined by the following parameters: $\sigma_{ij(\gamma)}(x_k, t)$, $\varepsilon_{ij(\gamma)}(x_k, t)$, $u_{i(\gamma)}(x_k, t)$ $(i, j, k = 1, 2, 3;\ \gamma = 1, 2)$ are the components of the stress and deformation tensors and displacement vectors of the plates, and $T_\gamma(x_k, t)$, $\chi_\gamma(x_k, t)$, $T_*(x_k, t)$, $\chi_*(x_k, t)$ are the temperature and specific strength of the internal heat sources in the plates and the layer, respectively. The boundary conditions written in terms of the stresses and conditions of heat exchange with external medium and with the heat conducting layer are specified at the end surfaces Ω_γ^+ and Ω_γ^-. The boundary conditions at the sides consist of mechanical and thermal conditions and depend on the way they are clamped and on the heat exchange conditions. The distribution of the displacements, velocities and temperature in the plates and the layer at the initial instant $t = 0$, are known.

The external forces and temperature fields acting on the plates cause them to bend towards each other, and the plates may come into contact. This is accompanied by the appearance of a previously unknown zone of dense contact $\Omega_e(t) = \Omega_1^- \cap \Omega_2^+$ changing with time, within which the contact forces of interaction $q_i(x_\alpha, t)$ $(i = 1, 2, 3;\ \alpha = 1, 2)$ appear and contact heat transfer occurs. The problem therefore consists of determining the stress-deformation state and the temperature fields within the plates, the region of dense (complete) contact

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$\Omega_e(t)$, and the contact interaction forces $q_i(x_\alpha, t)$.

2. ***Equations describing the thermodynamic state of the system.*** When considering the thermoelastic state of the plates, we start from the equations of thermoelasticity /1/

$$\mu\Delta u_{i(\gamma)} + (\lambda+\mu)\frac{\partial}{\partial x_i}\operatorname{div}\mathbf{u}_{(\gamma)} = \rho\frac{\partial^2 u_{i(\gamma)}}{\partial t^2} + (3\lambda+2\mu)\alpha_T\frac{\partial(T_\gamma - T_{0(\gamma)})}{\partial x_i} \tag{2.1}$$

$$\Delta T_\gamma + \frac{\chi_\gamma}{\lambda_T} = \frac{1}{a}\frac{\partial T_\gamma}{\partial t} + \frac{(3\lambda+2\mu)\alpha_T T_{0(\gamma)}}{\lambda_T}\frac{\partial}{\partial t}\operatorname{div}\mathbf{u}_{(\gamma)} \tag{2.2}$$

$$\left(\operatorname{div}\mathbf{u}_{(\gamma)} = \frac{\partial u_{1(\gamma)}}{\partial x_1} + \frac{\partial u_{2(\gamma)}}{\partial x_2} + \frac{\partial u_{3(\gamma)}}{\partial x_3}\right)$$

where Δ is the three-dimensional Laplace operator, λ and μ are the Lamé coefficients, ρ is the density of the plate material, α_T, λ_T and a are the coefficients of linear thermal expansion, thermal conductivity and thermal diffusivity, and $T_{0(\gamma)}$ is the initial temperature distribution corresponding to the undeformed state of the plates. We assume that the mechanical and thermal properties of the plates are the same.

All thermodynamic parameters within this section will depend on four variables $\mathbf{x} = (x_1, x_2, x_3)$, t. We shall omit these variables in order to shorten the notation, and we shall continue to use these abbreviations in what follows.

The temperature distribution within the layer is described by the equations of heat conduction

$$\Delta T_* + \chi_*/\lambda_* = a_*^{-1}\partial T_*/\partial t \tag{2.3}$$

where λ_* and a_* are the thermal conductivity and thermal diffusivity, respectively.

We will write the mechanical boundary conditions at the face surfaces of the plates in the form

$$\sigma_{i3(1)} = \sigma_{i3}^+, \quad \forall(\mathbf{x}) \in \Omega_1^+, \quad \sigma_{i3(2)} = \sigma_{i3}^-, \quad \forall(\mathbf{x}) \in \Omega_2^- \tag{2.4}$$

$$\sigma_{i3(1)} = \begin{cases} 0, & \forall(\mathbf{x}) \notin \Omega_e(t), \\ -q_i, & \forall(\mathbf{x}) \in \Omega_e(t), \end{cases} \quad \forall(\mathbf{x}) \in \Omega_1^-$$

$$\sigma_{i3(2)} = \begin{cases} 0, & \forall(\mathbf{x}) \notin \Omega_e(t), \\ -q_i, & \forall(\mathbf{x}) \in \Omega_e(t), \end{cases} \quad \forall(\mathbf{x}) \in \Omega_2^+$$

At the sides we have

$$u_{1(\gamma)} = u_{2(\gamma)} = u_{3(\gamma)} = 0, \ \forall(\mathbf{x}) \in \Gamma_\gamma \ \ (\gamma = 1, 2) \tag{2.5}$$

The boundary conditions on the outer surfaces are

$$T_1 = T_1^+, \ \forall(\mathbf{x}) \in \Omega_1^+, \ T_2 = T_2^-, \ \forall(\mathbf{x}) \in \Omega_2^- \tag{2.6}$$

and at the edges they are

$$\begin{gathered} \lambda_T\partial T_\gamma/\partial h + \alpha(T_\gamma - T) = 0, \ \forall(\mathbf{x}) \in \Gamma_\gamma \\ \lambda_*\partial T_*/\partial h + \alpha_*(T_* - T) = 0, \ \forall(\mathbf{x}) \in \Gamma_* \end{gathered} \tag{2.7}$$

where T is the temperature of the surrounding medium, and α and α_* are the heat transfer coefficients of the plates and the layer.

The case of other mechanical and thermal boundary conditions is treated in the same manner.

We shall assume that at the boundaries between the heat conducting layer and the plates ideal contact heat exchange takes place

$$T_1 = T_*, \ -\lambda_T\partial T_1/\partial h = \lambda_*\partial T_*/\partial h, \quad \forall(\mathbf{x}) \in \Omega_1^- \quad T_2 = T_*, \ \lambda_T dT_2/\partial h = -\lambda_*\partial T_*/\partial h, \quad \forall(\mathbf{x}) \in \Omega_2^+ \tag{2.8}$$

We shall write the initial conditions at $t = 0$ in the form

$$u_{i(\gamma)} = f_{i(\gamma)}, \quad \partial u_{i(\gamma)}/\partial t = q_{i(\gamma)} \quad (i = 1, 2, 3;\ \gamma = 1, 2)\ T_\gamma = T_\gamma^\circ, \quad T_* = T_*^\circ \tag{2.9}$$

and we will impose the following additional constraints on the normal displacements of the points of the surfaces Ω_1^- and Ω_2^+:

$$u_{3(2)} - u_{3(1)} \leqslant h_0 \tag{2.10}$$

We will assume that the surfaces Ω_1^- and Ω_2^+ are rough. Therefore, when the plates come into contact during their deformation and a region $\Omega_e(t)$ forms, we shall have contact with friction. Such a formulation leads to an analogue of the Signorini problem /2/ with friction. In the region of dense contact $\Omega_e(t)$ the mechanical parameters must satisfy the additional conditions (f is the coefficient of friction within the zone of contact)

$$
\begin{gathered}
\forall\,(x) \in \Omega_e(t),\ \ u_{3(2)} - u_{3(1)} = h \Rightarrow \\
\sigma_{33(1)} = \sigma_{33(2)} - q_3 \\
|\sigma_{\beta 3(\gamma)}| < f\,|q_3| \Rightarrow u_{\alpha(1)} = u_{\alpha(2)}\ (\alpha,\ \beta,\ \gamma = 1, 2)
\end{gathered}
\tag{2.11}
$$

We shall write the thermal conditions within the area of contact in the form

$$
q = \alpha_c(T_2 - T_1),\ \forall\,(\mathrm{x}) \in \Omega_e(t) \tag{2.12}
$$

where q is the heat flux passing across the area of contact and α_c is the contact thermal conductivity.

The analysis of the problem formulated here encounters considerable mathematical difficulties caused by the dimensions of the problem, as well as by its lack of linerity following from the contact conditions (2.10), (2.11). The problem can be partially simplified by making use of the fact that the bodies in question are plates and the gap between them is small. Let us expand the thermodynamic parameters describing the state of the system in series in Legendre polynomials $P_k(\omega)$ along the coordinate x_3, and set up the equations of the problem in question in terms of the coefficients of this expansion. As a result we obtain a system of equations which depend on only two spatial coordinates.

3. Reduction of the three-dimensional equations describing the thermodynamic state of the plates and the layer to two dimensions. Let us introduce a Cartesian system of coordinates with the origin at the middle of the surface of the undeformed, heat conducting layer. The x_1 and x_2 axes will lie in the middle surface, and x_3 will be perpendicular to it. The arguments accompanying the Legendre polynomials ω_i and ω_* should vary from -1 to +1, and must therefore be connected with x_3 by the following relations:

$$
\begin{gathered}
\omega_1 = (2x_3 - h_0 - h_1)/h_1,\ \ x_3 \in [h_0/2,\ h_1^+],\ \ h_1^+ = h_0/2 + h_1 \\
\omega_2 = (2x_3 + h_0 + h_2)/h_2,\ \ x_3 \in [h_2^-, h_0/2],\ \ h_2^- = -h_0/2 - h_2 \\
\omega_* = (2x_3 - h_*)/h,\ \ x_3 \in [h^-,\ h^+],\ \ h^+ = h_0/2 - \bar{u}_{3(1)} \\
h^- = -h_0/2 + \overset{+}{u}_{3(2)},\ \ h = h^+ - h^-,\ \ h_* = h^+ + h^-
\end{gathered}
\tag{3.1}
$$

where $\bar{u}_{3(1)}$ and $\overset{+}{u}_{3(2)}$ are the normal components of the displacement vectors of the points of the plates situated, respectively, on the surfaces Ω_1^- and Ω_2^+.

Multiplying Eqs.(2.1) by $(2k+1)h_\gamma^{-1}P_k(\omega_\gamma)$ $(k = 1, 2, \ldots, \infty)$ and integrating over x_3 from $h_0/2$ to h_1^+ for plate 1 and from h_2^- to $-h_0/2$, for plate 2 we obtain, after reduction

$$
\begin{gathered}
\Delta_2 u_{\alpha(\gamma)}^k + \frac{\lambda+\mu}{\mu}\frac{\partial}{\partial x_\alpha}\left(\frac{\partial u_{1(\gamma)}^k}{\partial x_1} + \frac{\partial u_{2(\gamma)}^k}{\partial x_2} + \frac{\lambda}{\mu}\frac{2k+1}{h_\gamma} \times \right. \\
\left.\left(\frac{\partial u_{3(\gamma)}^{k+1}}{\partial x_\alpha} + \frac{\partial u_{3(\gamma)}^{k+3}}{\partial x_\alpha} + \ldots\right)\right) - \frac{2k+1}{h_\gamma}\left[\frac{\partial u_{3(\gamma)}^{k-1}}{\partial x_\alpha} + \right. \\
\frac{\partial u_{3(\gamma)}^{k-3}}{\partial x_\alpha} + \ldots + \frac{2k-1}{h_\gamma}(u_{\alpha(\gamma)}^k + u_{\alpha(\gamma)}^{k+2} + \ldots) + \\
\left.\frac{2k-5}{h_\gamma}(u_{\alpha(\gamma)}^{k-2} + u_{\alpha(\gamma)}^k + u_{\alpha(\gamma)}^{k+2} + \ldots) + \ldots\right] + \\
\frac{2k+1}{2\mu h_\gamma}\Phi_{\alpha(\gamma)} - \frac{3\lambda+2\mu}{\mu}\alpha_T\frac{\partial T_\gamma^k - T_{0(\gamma)}^k}{\partial x_\alpha} = \frac{\rho}{\mu}\frac{\partial^2 u_{\alpha(\gamma)}^k}{\partial t^2} \qquad (\alpha = 1, 2) \\
\Delta_2 u_{3(\gamma)}^k + \frac{2k+1}{h_\gamma}\left(\frac{\partial u_{1(\gamma)}^{k+1}}{\partial x_1} + \frac{\partial u_{1(\gamma)}^{k+3}}{\partial x_1} + \ldots + \frac{\partial u_{2(\gamma)}^{k+1}}{\partial x_2} + \frac{\partial u_{2(\gamma)}^{k+3}}{\partial x_2} + \ldots\right) - \\
\frac{2k+1}{\mu h_\gamma}\left[\lambda\left(\frac{\partial u_{1(\gamma)}^{k-1}}{\partial x_1} + \frac{\partial u_{2(\gamma)}^{k-1}}{\partial x_2}\right) + \lambda\left(\frac{\partial u_{1(\gamma)}^{k-3}}{\partial x_1} + \frac{\partial u_{2(\gamma)}^{k-3}}{\partial x_2}\right) + \ldots\right. \\
+ (\lambda+2\mu)\frac{2k-1}{h_\gamma}(u_{3(\gamma)}^k + u_{3(\gamma)}^{k+2} + \ldots) + (\lambda+2\mu)\frac{2k-5}{h_\gamma}(u_{3(\gamma)}^{k-2} + u_{3(\gamma)}^k + \\
\left. u_{3(\gamma)}^{k+2} + \ldots) + \ldots\right] + \frac{2k+1}{2\mu h_\gamma}\Phi_{3(\gamma)} - (3\lambda+2\mu)\alpha_T(T_\gamma^{k-1} + T_\gamma^{k-3} + \ldots \\
- T_{0(\gamma)}^{k-1} - T_{0(\gamma)}^{k-3} - \ldots) = \rho\frac{\partial^2 u_{3(\gamma)}^k}{\partial t^2} \\
\Phi_{i(1)} = \sigma_{i3}^+ + (-1)^k q_i, \quad -\Phi_{i(2)} = q_i + (-1)^k \sigma_{i3}^-
\end{gathered}
\tag{3.2}
$$

where Δ_2 is a two-dimensional Laplace operator, and $u^k_{i(\gamma)}$ and $T_\gamma{}^k$ are the coefficient of the expansion of the components of the displacement vectors and the temperture of the plates in series in Legendre polynomials.

The equations of heat conduction transformed in this manner have the form

$$\Delta_2 T_\gamma{}^k - \frac{2k+1}{2h_\gamma}(Q_\gamma^{k-1} + Q_\gamma^{k-3} + \ldots) + \frac{\chi_\gamma{}^k}{\lambda} = \frac{1}{a}\frac{\partial T_\gamma{}^k}{\partial t} + \frac{(3\lambda+2\mu)}{\lambda}\alpha_T T^k_{0\gamma}\frac{\partial}{\partial t}\left[\frac{\partial u^k_{1(\gamma)}}{\partial x_1} + \frac{\partial u^k_{2(\gamma)}}{\partial x_2} - \frac{2k+1}{h_\gamma}(u^{k+1}_{3(\gamma)} + u^{k+3}_{3(\gamma)} + \ldots)\right] \tag{3.3}$$

Here Q_γ, $Q_\gamma{}^+$, $Q_\gamma{}^-$ and $Q_\gamma{}^k$ are the derivative of the temperature with respect to x_3, its values on the surfaces $\Omega_\gamma{}^+$ and $\Omega_\gamma{}^-$, and each term of its expansion in series in Legendre polynomials, respectively.

The functions $Q_\gamma{}^+$, $Q_\gamma{}^-$ and $Q_\gamma{}^k$ are found from the conditions of heat exchange at the face surfaces of the plates, and the recurrence relations

$$\frac{Q_\gamma^{k-1}}{2k-1} - \frac{Q_\gamma^{k+1}}{2k+3} = \frac{1}{h_\gamma}T_\gamma{}^k \tag{3.4}$$

When constructing the two-dimensional equations of heat conduction for the layer, we take into account the fact that its thickness changes when the plates are deformed. Let us multiply Eq. (2.3) by $(2k+1)h^{-1}P_k(\omega_*)$ $(k = 1, 2, \ldots)$ and integrate the resulting expression in x_3 from h^- to h^+. Carrying out the calculations and taking into account the fact that h^- and h^+ depend on x_α and t $(\alpha = 1, 2)$, we obtain the equations of heat conduction in the form

$$\begin{gathered}
\Delta_2 T_*{}^k + T^k_{*1}\Delta_* h + T^k_{*2}\Delta_* h_* + \frac{1}{h}(\nabla_2 T^k_{*1}\nabla_2 h + \nabla_2 T^k_{*2}\nabla_2 h_*) + \\
\frac{2k+1}{2}[T_*{}^+\Delta_* h^+ + (-1)^k T_*{}^-\Delta_* h^-] + \frac{1}{h}(\nabla_2 h \mathbf{Q}_1{}^k + \nabla_2 h_* \mathbf{Q}_2{}^k) + \\
\frac{2k+1}{2h}[Q_*{}^+ - (-1)^k Q_*{}^-] + \frac{2k+1}{2h}(Q_*^{k-1} + Q_*^{k-3} + \ldots) + \\
\frac{\chi_*{}^k}{\lambda_*} = \frac{1}{a}\left\{\frac{\partial T_*{}^k}{\partial t} + \frac{1}{h}\frac{\partial h}{\partial t}T^k_{*1} + \frac{1}{h}\frac{\partial h_*}{\partial t}T^k_{*2} - \right. \\
\left. - \frac{2k+1}{2h}\left[\frac{\partial h^+}{\partial t}T_*{}^+ - (-1)^k\frac{\partial h^-}{\partial t}T_*{}^-\right]\right\} \\
T^k_{*1} = (k+1)T_*{}^k + (2k+1)(T_*^{k-2} + T_*^{k-4} + \ldots),
\end{gathered} \tag{3.5}$$

$$\begin{gathered}
T^k_{*2} = (2k+1)(T_*^{k-1} - T_*^{k-3} + \ldots) \\
Q_{11}{}^k = (k+1)\left\{\frac{\partial T_*{}^k}{\partial x_1} - \frac{2k+1}{2h}F(T_*{}^+, T_*{}^-) + \frac{1}{h}\frac{\partial h}{\partial x_1}T^k_{*1} + \right. \\
\left.\frac{1}{h}\frac{\partial h_*}{\partial x_1}T^k_{*2}\right\} + (2k+1)\frac{\partial T_*^{k-2}}{\partial x_1} + \frac{\partial T_*^{k-4}}{\partial x_1} + \ldots + \frac{1}{h}\frac{\partial h}{\partial x_1}[(k-1)T_*^{k-2} + \\
(3k-6)T_*^{k-4} + \ldots] + \frac{1}{h}\frac{\partial h_*}{\partial x_1}[(2k-3)T_*^{k-3} + (4k-10)T_*^{k-5} + \ldots] - \\
\left.\left(\frac{2k-3}{2} + \frac{2k-7}{2} + \ldots\right)\frac{1}{h}F(T_*{}^+, T_*{}^-)\right\} \\
Q_{12}{}^k = (2k+1)\left\{\frac{\partial T_*^{k-1}}{\partial x_1} + \frac{\partial T_*^{k-3}}{\partial x_1} + \ldots + \frac{\partial}{\partial x_1}[kT_*^{k-1} + \right. \\
(3k-3)T_*^{k-3} + \ldots]\frac{1}{h}\frac{\partial h_*}{\partial x_1}[(2k-1)T_*^{k-2} + (4k-6)T_*^{k-4} + \ldots] - \\
\left.\left(\frac{2k-1}{2} + \frac{2k-5}{2} + \ldots\right)\frac{1}{h}F(T_*{}^+, T_*{}^-)\right\} \\
\Delta_* = \frac{\partial}{\partial x_1}\left(\frac{1}{h}\frac{\partial}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{1}{h}\frac{\partial}{\partial x_2}\right), \quad \nabla_2 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}, \\
\mathbf{Q}_2{}^k = (Q_{21}{}^k, Q_{22}{}^k) \\
\mathbf{Q}_1{}^k = (Q_{11}{}^k, Q_{12}{}^k), \quad F(T_*{}^+, T_*{}^-) = \frac{\partial h^+}{\partial x_1}T_*{}^+ - (-1)^k\frac{\partial h^-}{\partial x_1}T_*{}^-
\end{gathered}$$

The formulas for $Q_{21}{}^k$ and $Q_{22}{}^k$ are identical with the formulas for $Q_{12}{}^k$ and $Q_{11}{}^k$, provided that we replace the derivatives in x_1 in the latter by derivatives in x_2.

For the functions $Q_*{}^k$ we have recurrence relations analogous to (3.4), and the values of $T_*{}^+$, $T_*{}^-$, $Q_*{}^+$ and $Q_*{}^-$ are found from the conditions of heat exchange with the plates.

The thermodynamic parameters in (3.2) and (3.3) depend only on three variables (x_1, x_2), t.

4. ***Reduction of the boundary and contact conditions to have dimensions.*** The Eqs.(3.2), (3.3) and (3.5) are mutually dependent and their mutual dependence is governed by the structure of the equations, as well as by the conditions of heat exchange between the plates and the layer and of the contact between the plates.

Let us bring the initial, boundary and contact conditions into correspondence with the two-dimensional equations of thermoelasticity and heat conduction of the plates and the layer. To do this we shall expand the mechanical and thermal conditions on the ends (2.5), (2.7) and initial conditions (2.9), in series in Legendre polynomials, and write them in terms of the coefficients of the expansion.

We shall assume that the plates move under the action of the external load and the temperature field in such a manner that they do not come into contact with each other even at a single point, i.e. $\Omega_e(t) = 0$. The thermal conditions at the outer surfaces of the plates (2.6) and the conditions of their heat exchange with the heat conducting layer (2.8) will be written in the form (here and henceforth the summation will be carried out from $k = 0$ to $k = \infty$)

$$\begin{gathered}\Sigma T_1{}^k = T_1{}^+, \quad \forall (x_1, x_2) \in \Omega_1{}^+, \quad \Sigma(-1)^k T_2{}^k = T_2{}^-, \quad \forall (x_1, x_2) \in \Omega_2{}^- \\ \Sigma(-1)^k T_1{}^k = \Sigma T_*{}^k, \quad \lambda\Sigma(-1)^{k+1} Q_1{}^k = \lambda_*\Sigma Q_*{}^k, \quad \forall (x_1, x_2) \in \Omega_1{}^- \\ \Sigma(-1)^k T_*{}^k = \Sigma T_2{}^k, \quad \lambda_*\Sigma(-1)^{k+1} Q_*{}^k = \lambda\Sigma Q_2{}^k, \quad \forall (x_1, x_2) \in \Omega_2{}^+\end{gathered} \tag{4.1}$$

The above conditions and recurrence relations (3.4) together yield the functions $Q_\gamma{}^+$, $Q_\gamma{}^-$, $Q_\gamma{}^k$, $T_*{}^+$, $T_*{}^-$, $Q_*{}^+$, $Q_*{}^-$ and $Q_*{}^k$, which appear in (3.3) and (3.5). Thus we have constructed a mutually dependent system of quasilinear equations describing the thermoelastic state of the plates, and the heat exchange between them occurs through the heat-conducting layer without any contact between the plates.

If during the motion the plates should come in contact with each other and a region $\Omega_e(t)$ appears, then contact forces of interaction q_i will appear between them. The thermodynamic state of the system within the regions Ω_γ/Ω_e will be described by Eqs.(3.2), (3.3) and (3.5), and Ω_e will be described by its own system of equations which is much simpler than (3.2), (3.3) and (3.5). This is related to the fact that in this case the heat exchange takes place directly between the plates through the areas $\Omega_1{}^-$ and $\Omega_2{}^+$, and there is no longer any need for Eqs.(3.5).

The problem consists of solving the system of Eqs.(3.2), (3.3) determining the area of contact $\Omega_e(t)$ and the forces of interaction between the plates q_i within the area, under the constraints (2.11) which we shall transform as follows:

$$\begin{gathered}\forall (x_1, x_2) \in \Omega_e, \quad \Sigma u_{3(2)}^k - \Sigma(-1)^k u_{3(1)}^k = h_0 \Rightarrow \Sigma\sigma_{33(2)}^* = \Sigma(-1)^k \sigma_{33(1)}^k = -q_3 \\ |\Sigma\sigma_{\beta 3(2)}^k| < f|q_3|, \quad |\Sigma(-1)^k \sigma_{\beta 3(1)}^k| < f|q_3| \Rightarrow \\ \Sigma(-1)^k u_{\alpha(1)}^k = \Sigma u_{\alpha(2)}^k \quad (\alpha,\ \beta = 1, 2)\end{gathered} \tag{4.2}$$

We note that when the region of close contact Ω_e exists, the problem in question becomes considerably more complicated. This is due to the fact that in the contact-free regions (Ω_γ/Ω_e) the interdepenent quasilinear system (3.2), (3.3) and (3.5) remains valid, while in the region Ω_e we have the system of Eqs.(3.2), (3.3) with constraints (4.2) and unknown Ω_e and q_i. The problem consists of solving each system of equations and matching the solutions obtained on $\partial\Omega_e$.

5. ***Construction of the approximations.*** The system of Eqs.(3.2), (3.3), (3.5) has the advantage that the functions appearing in it depend on two spatial coordinates. It contains, however, an infinite number of equations. Reduction is used when carrying out the practical calculations, and the expansions of thermodynamic parameters in series in Legendre polynomials are truncated to contain a finite number of terms. We obtain the n-the approximation equations by varying the index k in (3.2)-(3.5), (4.1) and (4.2) from 0 to n.

Let us consider in greater detail the first-approximation equations. We shall write the equations of heat conduction of the plates (3.3) taking into account the conditions (4.1), in the form

$$\begin{gathered}\Delta_2 T_\gamma{}^\circ + \frac{3}{2h_\gamma{}^2} F_\gamma{}^\circ + \frac{\chi_\gamma{}^\circ}{\lambda} = \frac{1}{a}\frac{\partial T_\gamma{}^\circ}{\partial t} + \frac{3\lambda + 2\mu}{\lambda}\alpha_T T_{0(\gamma)}^\circ \frac{\partial}{\partial t}\left(\frac{\partial u_{1(\gamma)}^\circ}{\partial x_1} + \frac{\partial u_{2(\gamma)}^\circ}{\partial x_2} + \right. \\ \left.\frac{1}{h_\gamma} u_{3(\gamma)}^1\right) \\ \Delta_2 T_\gamma{}^1 + \frac{9}{h_\gamma{}^2} F_\gamma{}^1 - \frac{3Q_\gamma{}^\circ}{2h_\gamma} + \frac{\chi_\gamma{}^1}{\lambda} = \frac{1}{a}\frac{\partial T_\gamma{}^1}{\partial t} + \\ \frac{3\lambda + 2\mu}{\lambda}\alpha_T T_{0(\gamma)}^1 \frac{\partial}{\partial t}\left(\frac{\partial u_{1(\gamma)}^1}{\partial x_1} + \frac{\partial u_{2(\gamma)}^1}{\partial x_2}\right)\end{gathered} \tag{5.1}$$

$$F_1^\circ = T_1^+ + T_k^+ - 2T_1^\circ/h_1, \quad F_2^\circ = T_k^- + T_2^- - 2T_2^\circ/h_2$$
$$F_1^1 = T_1^+ - T_k^+ - 5T_1^1/(3h_1), \quad F_2^1 = T_k^- - T_2^- - 5T_2^1/(3h_2)$$
$$T_k^- = [(\lambda^2h^2 + \lambda\lambda_*hh_1)(27T_2^- + 54T_2^\circ + 90T_2^1) + \lambda_*^2h_1h_2(72T_*^\circ - 60T_*^1) + \lambda\lambda_*hh_2(54T_*^\circ - 90T_*^1 + 18T_1^\circ - 30T_1^1 + 9T_1^+)][81\lambda\lambda_*h(h_1+h_2) + 72\lambda_*^2h_1h_2]^{-1}$$

The expression for T_k^+ is obtained from the last formula by replacing h_1 by h_2, h_2 by h_1, T_2^- by T_1^+, T_2^1 by $-T_1^1$, T_*^1 by $-T_*^1$. The equations of heat conduction of the layer are

$$\Delta_2T_*^\circ + L_0(T_*^\circ, T_*^1) + \frac{3}{2h^2}(T_k^+ + T_k^-) = \frac{1}{a}\left\{\frac{\partial T_*^\circ}{\partial t} + \frac{1}{h}\frac{\partial h}{\partial t}T_*^\circ - \frac{1}{2h}\left[\frac{\partial h^+}{\partial t}T_*^+ - \frac{\partial h^-}{\partial t}T_*^-\right]\right\} \tag{5.2}$$
$$\Delta_2T_*^1 + L_1(T_*^\circ, T_*^1) + \frac{9}{h^2}(T_k^+ - T_k^-) = \frac{1}{a_*}\left\{\frac{\partial T_*^1}{\partial t} + \frac{2}{h}\frac{\partial h}{\partial t}T_*^1 + \frac{3}{h}\frac{\partial h_*}{\partial t}T_2^\circ - \frac{3}{2h}\left[\frac{\partial h^+}{\partial t}T_*^+ + \frac{\partial h^-}{\partial t}T_*^-\right]\right\}$$

where L_0 and L_1 are quasilinear differential operators.

We obtain the system of first-approximation equations for the plates from (3.2). The system separates into two independent subsystems. The first subsystem characterizes the motion of the plate within its plane, and the second within its plane of flexure.

We neglect the deformations of the plates within their planes, and in this case their motion can be fully described by the parameters $\overset{\circ}{u}_{3(\gamma)}$, $u^1_{1(\gamma)}$ and $u^1_{2(\gamma)}$. Models based on some or other hypotheses which simplify the system of equations of motion of the plates are often used instead of the equations of the first approximation. One of the simplest theories often used in solving practical problems is the theory based on the Kirchhoff hypotheses. The motion of the plates is described in this theory by the parameters $\overset{\circ}{u}_{3(\gamma)}$ called the flexures of middle surfaces and denoted by $\omega_{(\gamma)}$. Two other parameters of the theory of the first approximation are connected with it by the relations $u^1_{\alpha(\gamma)} = -h_\gamma\partial\omega_{(\gamma)}/\partial x_\alpha$ $(\alpha = 1, 2)$. The equations of motion of the plates in this case have the form

$$\Delta_2\Delta_2\omega_{(\gamma)} + (1+\nu)\alpha_T\Delta_2T^1_{(\gamma)} + \frac{2\rho h_\gamma}{D}\frac{\partial^2\omega_{(\gamma)}}{\partial t^2} = \frac{1}{D}(p-q) \tag{5.3}$$

where p is the normal load applied to the outer surfaces of the plates and q is the normal component of the contact pressure.

Even with these simplifiations the problem remains complicated. In the first place, this is caused by the presence of a quasilinear system of equations of heat conduction for the layer (5.2). Let us retain, in the expansion of the layer temperature in series in Legendre polynomials, a single term. Then the temperature field within the layer will be described by the first equation of (5.2). We have

$$T_k^- = [(9\lambda^2h + \lambda\lambda_*h_1)(3T_2^- + 6T_2^\circ + 10T_2^1) + \lambda\lambda_*h_2(3T_1^+ + 6T_1^\circ - 10T_1^1)]\{9[9\lambda^2h + \lambda\lambda_*(h_1+h_2)]\}^{-1}$$

The paramater T_*° is missing from this expression, and this means that the system of Eqs.(5.1), (5.3) is not connected with (5.2), and this simplifies the solution considerably.

Thus the contact problem in question is reduced, within the framework of the Kirchhoff model, to a system of Eqs.(5.1), (5.3) with condition of contact $\omega_2 - \omega_1 = h_0$.

We have obtained here the general equations of the problem of thermoelastic contact between plates through a heat-conducting layer based on the equations of coupled thermoelasticity. The equations of non-coupled dynamic, quasistatic or static thermoelastic contact problems can be obtained from them as special cases.

REFERENCES

1. BOLEY B.A. and WEINER J.H., Theory of Thermal Stresses. Wiley, N.Y.,1960.
2. DUVAUT G. and LIONS J.L., Inequalities in Mechanics and Physics. Springer, Berlin, 1976.
3. VEKUA I.N., Some General Methcds of Constructing Different Versions of the Theory of Shells. Nauka, Moscow, 1982.

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